Homework 5

Digital Communications (ECE 423)

Hani Mehrpouyan

QUESTION 1 (5 POINTS)

Suppose that two signal waveforms $s_1(t)$ and $s_2(t)$ are orthogonal over the interval $(0, T)$. A sample function $n(t)$ of a zero-mean, white noise process is cross correlated with $s_1(t)$, and $s_2(t)$, to yield

$$
n_1 = \int_0^T s_1(t)n(t)\,dt
$$

$$
n_2 = \int_0^T s_2(t)n(t)\,dt
$$

Prove that $\mathbb{E}(n_1n_2) = 0$.

QUESTION 2 (5 POINTS)

A binary digital communication system employs the signals

$$
s_0(t) = 0, \quad 0 \leq t \leq T
$$

$$
s_1(t) = A, \quad 0 \leq t \leq T
$$

for transmitting the information. This is called on-off signaling. The demodulator cross correlates the received signal $r(t)$ with $s_1(t)$ and samples the output of the correlator at $t = T$.

1) Determine the optimum detector for an AWGN channel and the optimum threshold, assuming that the signals are equally probable.

2) Determine the probability of error as a function of the SNR. How does on-off signaling compare with antipodal signaling?
QUESTION 3 (5 POINTS)

Three messages $m_1$, $m_2$, and $m_3$ are to be transmitted over an AWGN channel with noise power-spectral density $N_0/2$. The messages are

$$s_1(t) = \begin{cases} 1, & 0 \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

$$s_2(t) = -s_3(t) = \begin{cases} 1, & 0 \leq t \leq T/2 \\ -1, & T/2 \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

1) What is the dimensionality of the signal space?

2) Find an appropriate basis for the signal space (Hint: You can find the basis without using the Gram-Schmidt procedure).

3) Draw the signal constellation for this problem.

4) Derive and sketch the optimal decision regions $R_1$, $R_2$, and $R_3$.

5) Which of the three messages is more vulnerable to errors and why? In other words which of $P(\text{Error} | s_i \text{ transmitted}), i = 1, 2, 3$ is larger?

QUESTION 4 (5 POINTS)

In this chapter we showed that an optimal demodulator can be realized as:

- A correlation-type demodulator
- A matched-filter type demodulator

where in both cases $\phi(t), 1 \leq j \leq N$, were used for correlating $r(t)$, or designing the matched filters. Show that an optimal demodulator for a general $M$-ary communication system can also be designed based on correlating $r(t)$ with $s_i(t), 1 \leq i \leq M$, or designing filters that are matched to $s_i(t)$’s, $1 \leq i \leq M$. Precisely describe the structure of such demodulators by giving their block diagram and all relevant design parameters, and compare their complexity with the complexity of the demodulators obtained in the text.
**QUESTION 5 (5 POINTS)**

In a binary antipodal signalling scheme the signals are given by

\[ s_1(t) = -s_2(t) = \begin{cases} 
\frac{2A t}{T}, & 0 \leq t \leq T/2 \\
2A \left(1 - \frac{t}{T}\right), & T/2 \leq t \leq T \\
0, & \text{otherwise}
\end{cases} \]

The channel is AWGN and \( S_n(f) = N0/2 \). The two signals have prior probabilities \( p_1 \) and \( p_2 = 1 - p_1 \).

1) Determine the structure of the optimal receiver.
2) Determine an expression for the error probability.
3) Plot error probability as a function of \( p_1 \) for \( 0 \leq p_1 \leq 1 \).

**QUESTION 6 (5 POINTS)**

In an additive white Gaussian noise channel with noise power-spectral density of \( N0/2 \), two equiprobable messages are transmitted by

\[ s_1(t) = \begin{cases} 
\frac{At}{T}, & 0 \leq t \leq T \\
0, & \text{otherwise}
\end{cases} \]

\[ s_2(t) = \begin{cases} 
A \left(1 - \frac{t}{T}\right), & 0 \leq t \leq T \\
0, & \text{otherwise}
\end{cases} \]

1) Determine the structure of the optimal receiver.
2) Determine the probability of error.

**QUESTION 7 (5 POINTS)**

Consider the two 8-point QAM signal constellation shown in figure below. The minimum distance between adjacent points is \( 2A \). Determine the average transmitted power for each constellation assuming that the signal points are equally probable. Which constellation is more power efficient?