MIMO Systems in Wireless Networks

Michail Matthaiou
Signal Processing Group
Department of Signals and Systems
Chalmers University of Technology
12 April 2011
Personal background

- **1999-2004:** Diploma in Electrical and Computer Engineering, Aristotle University of Thessaloniki, Greece
- **2004-2005:** M.Sc. in Communication Systems and Signal Processing, University of Bristol, U.K.
- **2005-2008:** Ph. D. in Electrical Engineering, University of Edinburgh, U.K.
- **Sept. 2008-May 2010:** Post-doctoral research fellow at Technical University of Munich (TUM), Germany
- **July 2009-August 2009:** Visiting honorary fellow at University of Madison-Wisconsin, WI, USA
- **June 2010-present:** Assistant Professor at Chalmers University of Technology, Signal Processing Group, Sweden

**Areas of research:** MIMO systems, random matrix theory and multivariate statistics, performance analysis of fading channels, cooperative communications
Signal Processing group

Head: Prof. Mats Viberg

- 3 Full Professors, 2 Associate Professors, 2 Assistant Professors, 11 Ph.D students
- Statistical signal processing
- Image processing
- Computational electromagnetics
- Compressed sensing
- Cooperative communications
- MIMO systems
- Webpage: www.chalmers.se/s2/en
Brief history of wireless communications

• Early history of wireless communications:
  • 1861: Maxwell proposes theory of electromagnetic waves
  • 1887: Hertz demonstrates existence of such waves
  • 1895: Marconi builds first radio telegraph
  • 1921: Detroit Police Department instals 2 MHz mobile radio

• Recent history of wireless communications:
  • 1970: AT&T proposes first analog cellular telephone system
  • 1987: Winters introduces spatial multiplexing
  • 1989: ETSI accepts GSM standard
  • 1994: Paulraj and Kailath propose the use of multiple Tx/Rx antennas
  • 1995: Telatar derives MIMO channel capacity
  • 1996: Foschini proposes layered space-time coding
  • 1998: Alamouti introduces simple, full-rate orthogonal space-time code
Why MIMO?

- Higher data rates, improved reliability and coverage
- Very expensive spectrum licenses (increasing the operating bandwidth may not be a good idea!)
- Broadband over air
- Multimedia applications (video streaming, e-commerce...)
- Wireless internet access (WLAN, WiFi, WiMax)
- Wireless last-mile systems (home, office)
- Vehicular networks (vehicle-to-vehicle, vehicle-to-infrastructure)
- Short-range applications (indoor WiFi)
- Optical wireless communications
- Underwater communications (e.g. sonar)
- Radar applications (Enhanced beamforming performance)
Singe-Input Single-Output (SISO) systems

- Action of channel:
  - Block-constant flat-fading assumption:
    \[ y[m] = h x[m] + w[m] \]
  - Delay spread (frequency selectivity):
    \[ y[m] = \sum_{l} h[l] x[m-l] + w[m] \]
  - Delay/Doppler spread (time-frequency selectivity):
    \[ y[m] = \sum_{l} h[m,l] x[m-l] + w[m] \]

Channel causes linear distortion of input \( x[m] \) and adds noise \( w[m] \)
Singe-Input Multiple-Output (SIMO) systems

- Rx beamforming possible; Rx diversity gain
- Received signal (block-constant flat fading): $y[m] = h x[m] + w[m]$
- Channel described by $M_R \times 1$ vector

$$h = \begin{bmatrix} h_1 \\ \vdots \\ h_{M_R} \end{bmatrix}$$

$M_R$: number of receive antennas
Multiple-Input Singe-Output (MISO) systems

- Tx beamforming possible; Tx diversity gain and multiplexing gain
- Received signal (block-constant flat fading): \( y[m] = h^T x[m] + w[m] \)
- Channel described by \( M_T \times 1 \) vector

\[
h = \begin{bmatrix} h_1 \\ \vdots \\ h_{M_T} \end{bmatrix}
\]

\( M_T \): number of transmit antennas
Multiple-Input Multiple-Output (MIMO) systems

- Tx and Rx beamforming; diversity gain and multiplexing gain
- Received signal (block-constant flat fading):
  \[ y[m] = H x[m] + w[m] \]
- Channel described by \( M_R \times M_T \) matrix
  \[
  H = \begin{bmatrix}
  h_{1,1} & \cdots & h_{1,M_T} \\
  \vdots & \ddots & \vdots \\
  h_{M_R,1} & \cdots & h_{M_R,M_T}
  \end{bmatrix}
  \]
Overview of MIMO features

- **Array gain**
  
  ✓ SNR is increased by factor $M_R$ due to coherent combining at Rx

- **Spatial diversity**
  
  ✓ Means to combat fading by exploiting multiple uncorrelated replicas of the transmitted signal

Spatial diversity can be exploited at both sides of the MIMO radio link with the maximal diversity gain being $M_R \times M_T$
Overview of MIMO features – Part II

The most important MIMO feature is:

- **Multiplexing gain**
  - Transmit $M_T$ independent (orthogonal) data streams $\rightarrow$ increase capacity while leaving Tx energy and bandwidth unchanged
  - Maximum multiplexing gain is $\min\{M_R, M_T\}$
MIMO system model

- Transmitter:
  \[ b[m'] \xrightarrow{\text{coding, interleaving}} c[m''] \xrightarrow{\text{symbol mapping}} d[m'''] \xrightarrow{\text{space-time coding}} x[m] \xrightarrow{\text{radio-frequency part}} x(t) \]

- Channel (block-const. flat fading):
  \[ y[m] = H x[m] + w[m] \]

- Receiver:
  \[ y(t) \xrightarrow{\text{radio-frequency part}} y[m] \xrightarrow{\text{space-time decoding}} \tilde{d}[m'''] \xrightarrow{\text{demapping}} \tilde{c}[m''] \xrightarrow{\text{decoding, deinterleaving}} \tilde{b}[m'] \]

Joint design for improved performance (iterative receiver)
MIMO channel model

- **Block-constant flat-fading MIMO channel** (simplest model):
  \[
  y[m] = H x[m] + w[m]
  \]
  \[
  M_T \times 1 \quad M_R \times 1 \quad M_R \times M_T \quad M_R \times 1
  \]
  \[x[m]: \text{Tx vector} \quad y[m]: \text{Rx vector} \quad w[m]: \text{noise vector}\]

- **Channel matrix** (assumed block-constant and known at Rx):
  \[
  H = \begin{bmatrix}
  h_{1,1} & \cdots & h_{1,M_T} \\
  \vdots & \ddots & \vdots \\
  h_{M_R,1} & \cdots & h_{M_R,M_T}
  \end{bmatrix}
  \]
MIMO channel model - Part II

- Singular value decomposition (SVD) of $H$:

$$H = U \Sigma V^H$$

- Isometric matrices $U, V$ with $U^H U = V^H V = I_r$
- Positive diagonal matrix $\Sigma = \text{diag}\{\sigma_i\}_{i=1}^r$
- $r \leq \min(M_T, M_R)$ is rank of $H$ / singular values of $H$

- Obtain $r$ decoupled SISO channels:

$$y[m] = U \Sigma V^H x[m] + w[m]$$

$$U^H y[m] = \Sigma (V^H x[m]) + U^H w[m]$$

$$\tilde{y}[m] = \Sigma \tilde{x}[m] + \tilde{w}[m]$$
MIMO channel model—Part III

Result from previous slide:

\[
\tilde{y}[m] = \Sigma \tilde{x}[m] + \tilde{w}[m]
\]

⇒ Decoupled scalar equations (SISO channels)

\[
\tilde{y}_i[m] = \sigma_i \tilde{x}_i[m] + \tilde{w}_i[m], \quad i = 1, \ldots, r
\]
MIMO capacity

- MIMO system with $M_T$ transmit and $M_R$ receive antennas
- $r = \min(M_T, M_R)$
- Complex input-output relationship
  \[ y = Hx + n \]  
  (1)
- MIMO channel matrix response $H \in \mathbb{C}^{M_R \times M_T}$ with $H = U \Sigma V^H$
- Define $\Lambda = \Sigma^2 = \text{diag}\{\lambda_i\}_{i=1}^r$ with $\lambda_i = \sigma_i^2$ the eigenvalues of $HH^H$
- Note that $HH^H \in \mathbb{C}^{r \times r}$ is Hermitian, positive semi-definite and random.
MIMO capacity - Part II

- MIMO ergodic capacity assuming uniform power allocation [Telatar95]

\[
C' = \mathcal{E} \left[ \log_2 \left( \det \left( I_r + \frac{\text{SNR}}{N_t} \mathbf{H} \mathbf{H}^H \right) \right) \right] \\
= \mathcal{E} \left[ \log_2 \left( \det \left( I_r + \frac{\text{SNR}}{N_t} \mathbf{U} \Sigma \mathbf{V}^H \mathbf{V} \Sigma \mathbf{U}^H \right) \right) \right] \\
= \mathcal{E} \left[ \log_2 \left( \det \left( I_r + \frac{\text{SNR}}{N_t} \mathbf{U} \Sigma^2 \mathbf{U}^H \right) \right) \right]
\]

- Use the matrix property \( \det(I + AB) = \det(I + BA) \) to get

\[
C' = \mathcal{E} \left[ \sum_{i=1}^{r} \log_2 \left( 1 + \frac{\text{SNR}}{N_t} \lambda_i \right) \right]
\]

- High-SNR ergodic capacity (\( \text{SNR} \to \infty \))

\[
C^\infty = r \log_2 (\text{SNR}/N_t) + \frac{1}{\ln 2} \mathcal{E} \left[ \ln(\det(\mathbf{H} \mathbf{H}^H)) \right]
\]
MIMO capacity – Part III

• **SISO systems**
  • Fundamental limit on channel capacity set by signal-to-noise ratio (SNR)

• **MIMO systems**
  • Offer linear capacity growth with the minimum number of antennas

  ![Graph showing linear and logarithmic increase in capacity with number of antennas.](image)

  - Linear increase with *the minimum number of antennas*
  - Logarithmic increase with $M_R$
MIMO channel modeling

- Prevalent model: independent and identically distributed (i.i.d.) Rayleigh fading.

\[
H = H_w
\]  
(1)

where the entries of \(H_w\) entries are i.i.d. complex zero-mean, unit-variance random variables.

- Valid when the antenna spacings and/or the angular spreads are high enough to induce independent fading (zero spatial correlation) and a large number of multipaths impinge on the receiver from all directions.
- Simplifies extensively the mathematical manipulations and the performance analysis of MIMO technology.
- Rather unrealistic since spatial subchannels are rarely uncorrelated due to the limited angular spreads and array sizes.
MIMO channel modeling-Part II

Full channel correlation matrix:

\[ \mathbf{R}_H = E\{\text{vec}(\mathbf{H})\text{vec}(\mathbf{H})^H\} \]

\[ \mathbf{R}_H \in \mathbb{R}^{M_T M_R \times M_T M_R} \]

\[ \mathbf{h} = \begin{pmatrix} h_{11} & h_{12} & \ldots & h_{1 n} \\ h_{21} & h_{22} & \ldots & h_{2 n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{m1} & h_{m2} & \ldots & h_{mn} \end{pmatrix} \in \mathbb{C}^{M_R \times M_T} \]

- Elements of \( \mathbf{R}_H \) describe correlation between any pair of \( \mathbf{H} \) elements
- Full description of the channel matrix, if channel described by second-order statistics
- Elements of \( \mathbf{R}_H \) are difficult to interpret physically
- Full correlation matrix is very large \( \Rightarrow \) Find meaningful approximations of \( \mathbf{R}_H \)
MIMO channel modeling—Part II

- Kronecker correlation model

\[ H = R_R^{1/2} H_w R_T^{1/2} \]  \hspace{1cm} (2)

where \( R_T \in \mathbb{C}^{M_T \times M_T} \), \( R_R \in \mathbb{C}^{M_R \times M_R} \) denote the Hermitian, positive definite transmit and receive correlation matrix, respectively.

- Simple and allows for independent array optimization at transmitter and receiver
- Enforces the spatial correlation properties at both ends to be separable
- Creates artificial paths at the intersection of the real paths
- When large antenna arrays are used (improved angular resolution) the model’s performance is significantly impaired
MIMO channel modeling-Part III

Real joint spectrum

Modeled joint spectrum

Chase - the wireless future with Antenna Systems Excellence from CHALMERS
MIMO channel modeling—Part IV

- Weichselberger correlation model: Eigenvalue decompositions of $R_T$, $R_R$
  
  $R_T = U_T \Lambda_T U_T^H$
  
  $R_R = U_R \Lambda_R U_R^H$

- Coupling between the Tx/Rx is determined by the power coupling matrix $\Omega_{\text{weichsel}}$ whose positive and real-valued coefficients $\omega_{\text{weichsel}, m,n}$ specify the mean amount of energy coupled from the $n$-th transmit eigenvector to the $m$-th receive eigenvector

  $\Omega_{\text{weichsel}} = \mathcal{E}_H \left[ (U_R^H H U_T^*) \otimes (U_R^T H^* U_T) \right]$

  - The model enforces the spatial eigenbases at one side to be always the same for any spatial weight at the other side. The eigenvalues may differ.
  - Includes Kronecker model as a special case and yields better accuracy
  - Increased complexity
  - The multipath environment is occasionally not rendered accurately; this may lead to a rather blurred version of the APS.
MIMO channel modeling—Part V

Real joint spectrum

Modeled joint spectrum

Chase - the wireless future with Antenna Systems Excellence from CHALMERS
MIMO detectors

- ZF detection
  - Simplest linear detector
  - Project the received signal $y$ onto the subspace orthogonal to the one spanned by the vectors $h_1, \ldots, h_{k-1}, h_{k-1}, \ldots, h_{M_T-1}$
  - Cancels out interference but colors the additive noise term

$$y = Hx + n$$  \hspace{1cm} (5)

$$\text{ZF detection : } \hat{y} = x + H^\dagger n$$ \hspace{1cm} (6)

where $(\cdot)^\dagger$ is the pseudo-inverse of a matrix.

- Post-processing instantaneous SNR at the $k$-th ZF output:

$$\gamma_k \triangleq \frac{\text{SNR}}{M_T \left[ (H^H H)^{-1} \right]_{kk}}, \quad 1 \leq k \leq M_T$$ \hspace{1cm} (7)

where $[\cdot]_{kk}$ returns the $k$-th diagonal element of a matrix.
MIMO detectors-Part II

Schematic representation of the projection operation: $\mathbf{y}$ is projected onto the subspace orthogonal to $\mathbf{h}_1$ in order to demodulate stream 2.
MIMO detectors-Part III

- Minimum mean-square error (MMSE) detection
  - More complicated than the ZF detector
  - The detector is formulated in order to minimize the mean-square error cost function

\[
W_{\text{mmse}} = \arg \min_G \mathcal{E} \left[ \| x - Gy \|^2 \right]
\] (1)

\[
W_{\text{mmse}} = \sqrt{\frac{M_T}{P}} H^H \left( HH^H + \frac{M_T}{\text{SNR}} I_{M_T} \right)^{-1}
\] (2)

MMSE detection: \( \hat{y} = W_{\text{mmse}}(hx + n) \) (3)

- Post-processing instantaneous SNR at the \( k \)-th MMSE output:

\[
\gamma_k \triangleq \frac{1}{\left( I_{M_T} + \frac{\text{SNR}}{M_T} HH^H \right)^{-1}} - 1, \quad 1 \leq k \leq M_T
\] (4)
MIMO advantages

• Capacity scales linearly with number of antennas
  Channel knowledge/estimation at Rx needed

• MIMO offers potential for
  ✓ larger data rate
  ✓ larger spectral efficiency
  ✓ larger number of users
  ✓ improved range/coverage
  ✓ better interference suppression
  ✓ better quality of service (QoS), lower bit-error rate (BER)
  ✓ lower Tx power
MIMO disadvantages

• Hardware complexity:
  ✓ Each antenna needs a radio-frequency (RF) unit
  ✓ Powerful digital signal processing (DSP) unit required

• Software complexity:
  ✓ Most signal processing algorithms are computationally intensive

Power consumption:
  ✓ Battery lifetime of mobile devices
  ✓ Thermal problems

Antennas:
  ✓ Antenna spacing (electromagnetic mutual coupling-e.g. mobile handsets)
  ✓ RF interference and antenna correlation
Applications and Standards

- **3GPP: UMTS with MIMO enhancements**
  - HSDPA: Enhanced 3G mobile telephony protocol, Allows UMTS-based networks to have higher data rates (14.4 Mbits/s) - In the first week of May 2010, Indosat (Indonesia) launched the first HSPA+ 42Mbits/s network
  - HSPDA+: Combined with MIMO, 64 QAM and can offer up to 84.4 Mbits/s
  - The second phase of HSDPA is named HSDPA Evolved

- **3GPP: LTE with MIMO enhancements**
  - First publicly available LTE-service was launched by TeliaSonera in Stockholm and Oslo in December 2009, followed by operators in the US and Japan
  - **LTE-advanced** aims to use 8x8 MIMO and 128 QAM and promises to deliver 1Gbits/s at fixed speeds and 100Mbits/s to mobile users

UMTS: universal mobile telephone system  
HSDPA: High Speed Downlink Packet Access  
LTE: long term evolution  
OFDM: orthogonal frequency-division multiplexing
Applications and Standards—Part II

• **WLAN according to IEEE 802.11n (WiFi)**
  - Significant increase in the data rate compared to the previous standards to (i.e. 802.11a/g) → High-throughput: 600 Mbits/s
  - Can potentially allow for 4x4 MIMO configurations and 40 MHz channels (20 MHz in previous standards)

• **BRAN according to IEEE 802.16a,e,m (WiMax)**
  - IEEE 80.16 standard represents a series of Wireless Broadband standards
  - WiMAX (IEEE 802.16e-40Mbits/s) while IEEE 802.16m up to 1Gbits/s
  - Wireless last mile / LTE competitor
  - OFDM-based

✓ MIMO techniques used: spatial multiplexing, ST coding, precoding

WLAN: wireless local-area network  
BRAN: broadband radio access network  
WiMAX: Worldwide Interoperability for Microwave Access
Future research directions

- **Large MIMO**: Hundreds of low-power antennas (1mW) placed on a BS → potential for significant performance gains
- **MIMO relaying networks**: Combination of cooperative and MIMO technologies for increased capacity, reliability and coverage
- **Cognitive radio**: Detect “holes” in the expensive spectrum
- **Heterogeneous networks**: Combination of macrocells with pico and femtocells (increased indoor coverage and power efficiency)
- **Multicell MIMO**: multiple BSs each equipped with multiple antennas—> Main challenge is interference mitigation
- **Estimation of practical impairments**: in practical communication systems, performance is affected by several factors (timing offset, frequency offset and phase shift) that need to be estimated and compensated
Conclusions

- Review of MIMO technology
  - Main features (array gain, diversity, spatial multiplexing)
  - MIMO channel model (Parallel SISO subchannels)
  - MIMO capacity (Linear capacity growth with the minimum number of transmit/receive antennas)
  - MIMO channel modeling (i.i.d. Rayleigh, Kronecker and Weischelberger)
  - MIMO detectors (ZF and MMSE)
  - MIMO advantages, MIMO disadvantages

- MIMO applications and standardization

- Future research directions

* Some of the ideas covered in this seminar originate from the ”MIMO Communications” course given at Technical University of Vienna, Austria (Prof. Christoph F. Mecklenbräuker)
MIMO Literature

• A. Sibille et al., “MIMO: From Theory to Implementation,” Elsevier, 2011
If you are interested in a M.Sc. Thesis on MIMO Systems please contact me on micmat@chalmers.se (6th floor of E-building)

Thank you for your attention!