Lab 3 – Finding the value of an unknown capacitor

By Henry Lin, Hani Mehrpouyan

First, get familiar with the oscilloscope. It is a powerful tool for looking at waveforms. This allows voltages to be measured as a function of time.

1. Measure the voltage from the power supply. You should see a fairly constant line.
2. Now, connect the function generator to the oscilloscope. Make a 1kHz sine wave with the function generator. Measure the period of the sine wave (the time from peak to peak). Calculate the frequency from the period. Measure the amplitude.
3. Try changing the amplitude and frequency of the waveform. Change the waveform from sine to triangle. Change from triangle to square. For the different waveforms, find the amplitude and frequency.

Make sure to take pictures of the plots and oscilloscope screen. To take pictures of the oscilloscope screen, insert a flash drive into the USB port. Then, click Save/Recall -> Change Print Button to Save All to Files. You can select the folder and also change action to save image under different format. Then click the print button to save. The second way to take pictures of the oscilloscope is to use the OpenChoice Desktop software on the computer. The oscilloscope needs to be on before you turn on the computer.

The basic equation for capacitors is \( Q = CV \), where \( Q \) is the charge, \( C \) is the capacitance, and \( V \) is the voltage. Make sure the orientation of the capacitor is correct. The negative side of the capacitor has a shorter lead and sometimes an arrow pointing to it.

A. Finding the capacitance in parallel

![Figure 1. Setup for parallel capacitors test.](image)

Set up the circuit in Figure 1. From Figure 1, we have two capacitors. Capacitor 1 (C1) is unknown and
capacitor 2 (C2) is a decade capacitance box. Use 1µF, 2µF, 3µF, and 4µF on the decade capacitance box as four different values of C2. For parallel capacitors, \( C_{eq} = C_1 + C_2 \). When the switch is in position 1, capacitor 1 can be described by the following equation. \( Q_1 = C_1 V_0 \) while capacitor 2 is discharged. After the switch goes to position 2, the total charge \( Q = Q_1 + Q_2 \). But, \( Q_2 = 0 \) since it is discharged. Therefore \( Q = Q_1 \). This becomes \( C_{eq} V = C_1 V_0 \). Plugging in for \( C_{eq} \) gives \( (C_1 + C_2) V = C_1 V_0 \). This gives \( V = V_0 C_1 / (C_1 + C_2) \). Rearranging the equation becomes \( V_0 / V = C_2 / C_1 + 1 \). The value of C2 can change. As C2 changes, V can also change according to the equations.

1. With the DPDT knife switch, make sure all 6 wires are touching each metal part. Keep the switch in position 1. C2 is discharged while C1 is charged to V0. Try using a V0 of 15V from the power supply. Read the voltage on the oscilloscope. On the oscilloscope, use 5V/div and 500ms/div. Move the switch to position 2. C1 and C2 are in parallel. Right after you move the switch, the voltage should have a sudden drop. Read the voltage across the capacitors in parallel on the oscilloscope right after it drops. After some time, the voltage will quickly drop to zero. Hit “Run/Stop” on the oscilloscope to stop the screen from changing. Then, use “Cursor” on the oscilloscope to read the voltage.
2. Repeat the measurement several times to make sure it is correct. Also, repeat for different values of C2.
3. Make a plot of \( V_0 / V \) vs C2. Find the value of C1 from the plot. To do this in LibreOffice Calc, enter the x values in one column. Enter the y values in the next column. Then, insert chart from the top menu. For chart type, choose XY (Scatter). Then go to “Insert” -> “Trend Lines” -> Select “Linear” and Check “Show Equation”. For this part, C2 are the x values and \( V_0 / V \) are the y values. Then, the slope is equal to \( 1 / C_1 \).
4. Find the unknown capacitance of both small capacitors.

B. Finding the capacitance in series

Set up the circuit in Figure 2. The two capacitors are not charged in position 1. Capacitor 1 (C1) is unknown and capacitor 2 (C2) is a decade capacitance box. Use 1µF, 2µF, 3µF, and 4µF on the decade capacitance box as four different values of C2. On position 1, the voltage should be zero across the capacitors since the capacitors are discharged. When switched to position 2, you should see a sudden jump in voltage. When switched to position 2, record the voltage V1 across C1 right after it jumps.
Repeat several times. Do the same for voltage V2 across C2 right after it jumps. For each C2, V1 and V2 should add up to V0 (about 15V). Repeat for different values of C2.

For capacitors in series, \( C_{eq} = \frac{C1 \cdot C2}{(C1+C2)} \), \( Q1=Q2=Q \), and \( V0=V1+V2 \).
\[
\begin{align*}
Q &= C_{eq} \cdot (V1+V2) \\
C1 \cdot V1 &= C_{eq} \cdot V0 \\
C1 \cdot V1 &= (C1 \cdot C2) \cdot V0 / (C1+C2) \\
V1 &= C2 \cdot V0 / (C1+C2) \\
V2 &= C1 \cdot V0 / (C1+C2)
\end{align*}
\]

After measuring voltages V1 and V2, find the unknown capacitance. Rearranging the last two equations will give you the equation for the unknown capacitor. You should get similar values for \( C1 \) from both equations. Use the same two capacitors before in part A and find both unknown capacitance again.

\[
\begin{align*}
C1 &= \frac{(C2 \cdot V0 - V1 \cdot C2)}{V1} \\
C1 &= \frac{V2 \cdot C2}{(V0-V2)}
\end{align*}
\]

C. Finding the capacitance in a RC circuit with DC source

So far, no resistor was used, so the discharge from the capacitors was fairly quick. Set up the following circuit. Use the biggest capacitor. At first, the capacitor has a constant voltage across it from the dc source.

\[
\text{Disconnect the voltage source and discharge the capacitor. If you have a resistor, the time dependent equation for the voltage across a discharging capacitor is: } V_{\text{cap}}(t) = V_0 e^{-\frac{t}{RC}}. \text{ We are using a 10000 ohm resistor. Observe the voltage across the capacitor. Use 1sec/div.}
\]
1. Try using an initial voltage of 15V. Measure the time it takes to discharge to 80%, 60%,… of the initial voltage.
2. To do step 1, use two cursors. Put one cursor at the beginning when the voltage starts to drop. Move the second cursor around. Read the voltage on the second cursor and Δt.
3. Do the above ten times.
4. Average the results.
5. Plot [-ln(V/V0)] against time.
6. Draw a straight line. From the slope get the value of C. The slope should be equal to 1/(R*C).

D. Finding the capacitance in a RC circuit with AC source (1st method)

What happens if we use an AC source instead of a DC source? Set your function generator to create a sine wave with a voltage amplitude of a nice round number like 3V. You may want to adjust your frequency later, but start at about 1 Hz. Use a resistor of around 10000 ohms.

Next you will test the relationship $X_c = \frac{1}{\omega_0 C}$ by observing a sinusoidally driven RC circuit using many different driving frequencies. As you increase the driving frequency, the amplitude of the resistor voltage will increase because the total circuit impedance is decreasing, i.e. $V_{\text{resistor amplitude}} = R \frac{V_{\text{source amplitude}}}{Z}$.

Meanwhile, as the driving frequency increases, the capacitor amplitude decreases. This makes sense because the resistor and the capacitor are the only two components in the circuit other than the source. Since the voltages across both must add up to the source voltage at any instant in time, if the voltage amplitude of one increases, then the other must decrease.
Therefore, there must be some specific driving frequency when the amplitude of the resistor voltage matches the capacitor voltage: \( V_{\text{resistor amplitude}} = V_{\text{capacitor amplitude}} \) for a specific angular driving frequency \( \omega_{D, \text{match}} \).

Realizing that \( V_{\text{resistor amplitude}} = \frac{R}{Z} V_{\text{source amplitude}} \) and \( V_{\text{capacitor amplitude}} = \frac{X_C}{Z} V_{\text{source amplitude}} \), setting these two voltages equal when at the matching angular driving frequency \( \omega_{D, \text{match}} \) you get \( \frac{X_C}{Z} V_{\text{source amplitude}} = \frac{R}{Z} V_{\text{source amplitude}} \), which simplifies to \( X_C = R \). In other words, the voltage across the capacitor equals the voltage across the resistor if their "resistances" are equal, which kind of makes sense.

The “single measurement” method for finding the capacitance of an unknown capacitor makes use of the previous equation, \( X_C = R \). All you need to do is adjust the driving frequency of your circuit until the capacitor voltage amplitude and the resistor voltage amplitude are equal. Then use \( X_C = R \) (substituting \( X_C = \frac{1}{\omega_{D, \text{match}} C} \)) for the specific \( \omega_{D, \text{match}} \) to find the capacitance.

1. You do not need to measure with the probes across the resistor. Only use the probes to measure across the capacitor and across the total voltage with the oscilloscope. Do not set up middle ground like in the figure or you cannot measure the voltage across the capacitor. Connect the function generator and capacitor to the same ground.
2. Use the “Math” button to get subtraction and (Ch1-Ch2). Ch1 is the total voltage. Ch2 is the voltage across the capacitor. The Math waveform is the voltage across the resistor. Use 1V/div for Ch1, Ch2, and Math.
3. Use “Measure” on the oscilloscope to find the peak-to-peak voltage of the capacitor and the peak-to-peak voltage of the resistor. If it is not giving a value, go to Trig Menu and change the mode from Auto to Normal or try hitting Autoset.
4. Starting from 1 Hz, adjust the frequency until the peak-to-peak voltage of the capacitor is equal to the peak-to-peak voltage of the resistor. Enter this frequency and \( X_C = R = 10000 \text{ ohms} \) into the equation \( C = \frac{1}{2 \pi f X_C} \).
5. Do this for both small capacitors.

E. Finding the capacitance in a RC circuit with AC source (2nd method)

The “multiple measurements” method for finding an unknown capacitance is more involved, but more accurate as it involves multiple measurements. The voltage amplitudes of the sinusoidally driven RC are:

\[
V_{\text{resistor amplitude}} = \frac{R}{Z} V_{\text{source amplitude}}
\]

and
\[ V_{\text{capacitor amplitude}} = \frac{X_C}{Z} V_{\text{source amplitude}}. \]

Dividing these two equations gives

\[
\frac{V_{\text{capacitor amplitude}}}{V_{\text{resistor amplitude}}} = \left( \frac{X_C \cdot \frac{V_{\text{source amplitude}}}{Z}}{R \cdot \frac{V_{\text{source amplitude}}}{Z}} \right) = \frac{X_C}{R}.
\]

Therefore,

\[ X_C = R \frac{V_{\text{capacitor amplitude}}}{V_{\text{resistor amplitude}}}. \]

In order to experimentally determine \( C \) for your capacitor, simply combine the last equation with the definition \( \chi_C = \frac{1}{\omega_{\text{drive}} C} \) and rearrange:

\[ \frac{1}{R} \frac{V_{\text{resistor amplitude}}}{V_{\text{capacitor amplitude}}} = C \omega_D. \]

\[ \frac{1}{R} \frac{V_{\text{resistor amplitude}}}{V_{\text{capacitor amplitude}}} = C \omega_D \]

looks like a weird arrangement for this equation, but if you think of \( y=mx \), then you see that if you graph \( \frac{1}{R} \frac{V_{\text{resistor amplitude}}}{V_{\text{capacitor amplitude}}} \) vs. \( \omega_D \), you should obtain a linear graph with a slope equal to \( C \).

1. Like in part D, use the probes to measure the peak-to-peak voltage of the capacitor on the oscilloscope for different frequencies. Also, measure with probes across the
total voltage on the oscilloscope.

2. Use the “Math” button to get subtraction and (Ch1-Ch2). Ch1 is the total voltage. Ch2 is the voltage across the capacitor. The Math waveform is the voltage across the resistor.

3. Use “Measure” on the oscilloscope to find the peak-to-peak voltage of the capacitor and the peak-to-peak voltage of the resistor. If it is not giving a value, go to Trig Menu and change the mode from Auto to Normal or try hitting Autoset.

4. Make a plot of $\frac{1}{R} \frac{V_{\text{resistor amplitude}}}{V_{\text{capacitor amplitude}}}$ VS. $\omega_0$. $\omega_0$ is equal to $2 \pi$*frequency. R is still 10000 ohms.

5. Do this for both small capacitors.

F. Finding the capacitance from the physical characteristics

The capacitance can also be found by the equation

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d}$$

where A is the area of overlap of the two plates; $\varepsilon_r$ is the relative static permittivity, $\varepsilon_0$ is the electric constant ($\varepsilon_0 \approx 8.854 \times 10^{-12}$ F m$^{-1}$); and $d$ is the separation between the plates.